Origin of end-of-aging and subaging scaling behavior in glassy dynamics

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Linear response functions of aging systems are routinely interpreted using the scaling variable $t_{\rm obs}/t_w^{\mu}$, where t_w is the time at which the field conjugated to the response is turned on or off, and where $t_{\rm obs}$ is the "observation" time elapsed from the field change. The response curve obtained for different values of t_w are usually collapsed using values of μ slightly below one, a scaling behavior generally known as *subaging*. Recent spin glass thermoremanent magnetization experiments have shown that the value of μ is strongly affected by the form of the initial cooling protocol [G. F. Rodriguez *et al.*, Phys. Rev. Lett. **91**, 037203 (2003)], and even more importantly [G. G. Kenning *et al.*, Phys. Rev. Lett. **97**, 057201 (2006)], that the t_w dependence of the response curves vanishes altogether in the limit $t_{\rm obs} \gg t_w$. The latter result shows that $t_{\rm obs}/t_w^{\mu}$ scaling of linear response data cannot be generally valid, thereby casting some doubt on the theoretical significance of the exponent μ. In this work, a common mechanism is proposed for the origin of both subaging and end of aging behavior in glassy dynamics. The mechanism combines real and configuration space properties of the state produced by the initial thermal quench which initiates the aging process.

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I. INTRODUCTION

In glassy systems, a thermal quench initiates a so called aging process, whereby physical observables, e.g., the energy, slowly change as a function of "age," a term conventionally denoting the time t elapsed from the quench. In nonstationary processes, conjugated linear response and correlations functions generally depend on two time arguments, e.g., the system age t, and its value t_w at the moment where the field is switched on, or off. However, both functions appear to actually depend on a single scaling variable, namely, $t_{\rm obs}/t_w^{\mu}$, where $t_{\rm obs} \overset{def}{=} t - t_w$ [1–3]. The "observation" time $t_{\rm obs}$, (widely denoted by the symbol t in the literature) is often used in lieu of the system age as an independent time variable. The exponent μ is generally near one, and the terms *superaging*, *subaging*, and *pure* or *full* aging are used to discriminate between cases with $\mu > 1$, $\mu < 1$, and $\mu = 1$, respectively. Subaging (henceforth, SA) has mainly been observed in spin-glass thermoremanent magnetization (TRM) data [2,4–6], usually after subtracting a "stationary" term, which describes the response immediately after the field is cut.

As a quench is unavoidably carried out at a finite cooling rate, the point $t=0$ on the time axis eludes a sharp definition in an experimental setting. Possibly as a consequence thereof, the scaling form of response functions is, in spite of a protracted debate [4,5,7–11], only partially understood. E.g., the physical significance of μ (and of the additional time scale it brings along) is unclear, even more so in the light of the recent discovery that the TRM loses its t_w dependence in the limit $t_{\rm obs}/t_w \to \infty$ [11]. This last observation falsifies the long-held hypothesis that SA could be a scaling form generally applicable to linear response functions. Coined in [11], the term "end-of-aging" (EOA) is also adopted here. It describes that, for large values of t/t_w, the t_w dependence of the TRM disappears and is replaced by a simple logarithmic time decay. The term EOA should not be construed as contradicting the fundamental unity of aging dynamics, which subtends our description.

In this work, an expression for the TRM decay, Eq. (8), is derived, which, depending on the value of the model parameter x, interpolates between pure aging (PA), SA, and EOA behavior. Our derivation combines well known configuration and real space properties of spin glasses and supplements them with a model assumption regarding the spatial heterogeneity of the system configuration at the beginning of the isothermal aging process. Our analytical expression for the TRM decay is derived by averaging the magnetic response of independent domains over a suitable distribution characterizing their initial state. Even though the ratio $t_{\rm obs}/t_w^{\mu}$ nowhere appears in the treatment, the TRM decay curves produced by our Eq. (8) can, in accordance with standard practice, be *empirically collapsed* in the SA regime using $t_{\rm obs}/t_w^{\mu}$ scaling.

II. LINEAR RESPONSE AND DOMAIN HETEROGENEITY

Thermalized domains whose linear size grows with the thermal correlation length [12] are ubiquitous in aging systems with short range interactions. Real space scaling descriptions [13] rely on their properties to account for several dynamical properties, e.g., so-called "chaos" effects. In a physics context, hierarchies were brought to the fore by the Parisi equilibrium solution of a mean field model [14], a solution, which has inspired manifold hierarchical models of complex dynamics, e.g., [15]. Hierarchical dynamics is a generic feature of complex systems [16–18], and a feature which is complementary, not antithetic, to real space descriptions.

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Hierarchical models, see, e.g., [19–21], assume that nested ergodic components [22] exist in configuration space. In simple cases, the solution of the pertinent master equations shows PA behavior [20]. For thermally activated dynamics, this result can be heuristically explained as follows: any ergodic component of the hierarchy is at time t indexed by a real valued "dynamical barrier" $b(t)$. On the (Arrhenius) time scale $t_b(t)=C\exp[b(t)/T]$ the component is near a state of internal thermal equilibrium. The constant C is the *fluctuation time*, i. e. the smallest relevant time scale of the dynamics. Consider now a system starting in a component with vanishing initial barrier $b(t=0)\approx 0$. After aging isothermally for a time $t=t_w$, the component is characterized by a barrier $b(t_w)\propto T\ln(t_w/C)$. On scales $t_{\rm obs}<t_w$ the dynamics has character of quasiequilibrium fluctuations within the same component, while off-equilibrium processes involving larger components take over for $t_{\rm obs}>t_w$. The age t_w, hence, separates the two dynamical regimes observed for aging systems, and constitutes the dominant time scale for $t>t_w$. From this observation, PA heuristically follows. A mathematically more precise route leading to the same conclusion relies on the fact that, within a hierarchy indexed by energy barriers, only thermal energy fluctuations of *record* magnitude are able to trigger irreversible changes of ergodic component, or *quakes*. Since the statistics of record-sized fluctuations in a stationary series is known analytically [23,24], assuming that all physical changes are statistically subordinated to the quakes leads to *record dynamics* [24–26] and to analytical formulas, such as Eq. (3), for one and two-point averages of aging processes. In spite of their simplicity, the above ideas rationalize a large amount of experimental evidence [10,27,28]. Yet, they neither account for SA nor for EOA scaling behaviors. In order to do so, the spatial and temporal heterogeneity of spin glasses, experimentally demonstrated by Chamberlin [29], must be properly taken into account.

In the present model, independently relaxing domains of a glassy system are all endowed with the same type of hierarchically structured configuration space. Nonetheless, their respective contributions to the overall linear response are different (albeit related) functions of time. The assumption is that domains find themselves in states characterized by different dynamical barriers at the end of the initial quench, or equivalently, at the beginning of the isothermal aging process.

The physical mechanism behind the difference is likely related to the way in which the cooling process proceeds near the glass transition temperature, see, e.g., [6,30]. Here, the spatial heterogeneity is heuristically described by a distribution of initial barriers $P(b)$. We checked that a flat distribution supported in the interval $(0,b_M^*)$ and an exponential distribution with average b_M^* lead to similar behaviors. The finiteness of the first moment of $P(b)$ seems hence to be the crucial feature. Since the exponential form leads to simpler closed form expressions, this form is chosen for mathematical convenience.

In summary the initial state of the aging process feature domains described by the dynamical barriers distribution,

$$P(b)=\frac{1}{b_M^*}\exp(-b/b_M^*). \tag{1}$$

Let t_M^* be the Arrhenius time associated to b_M^*. As we shall see, the quantity

$$x\overset{\rm def}{=}\frac{T}{b_M^*}=\frac{1}{\ln(t_M^*/C)}, \tag{2}$$

controls all deviations from PA behavior. Note that $x>0$, that $x\to 0$ for $t_M^*\to\infty$, and that $x\to\infty$ for $t_M^*\downarrow C$.

III. PURE AGING APPROXIMATION OF TRM DATA

In the present theory, the additive contribution to the magnetic response of a single domain is given by Eq. (5), where the function $M_0(t/t_w)$ describes the TRM of a domain initially in a state having a vanishing dynamical barrier.

The functional form chosen for M_0 reflects that record dynamics is a homogeneous stochastic process in the single "time" variable $\log(t/t_w)$. By standard arguments, all moments of the process, including the average response, admit eigenvalue expansions where $\log(t/t_w)$ replaces time. The generic term in such expansions is proportional to $(t/t_w)^{\lambda_k}$, where λ_k is the k-th relaxation eigenvalue. In practice, the expansion can be truncated after few terms and, as shown graphically in the Appendix, two terms (one term less than in [10]) already provide an acceptable parameterization of the TRM decay in the PA approximation.

Summarizing, the PA scaling ansatz for the TRM can be written as

$$M_0(z)=M_I+\eta(z-1)\sum_{k=1}^{2}\frac{a_k}{\lambda_k}(z^{\lambda_k}-1), \tag{3}$$

where $z=t/t_w$, and where η is the Heaviside step function, and M_I is the "initial" value of the TRM, which for simplicity is treated as a parameter. According to the formula, the TRM remains constant and equal to M_I until the magnetic field is cut. The notation a_k/λ_k for the prefactor is chosen to simplify the form of the rate of magnetization change, which reads

$$r_{\rm TRM,0}(t,t_w)=\frac{1}{t}\sum_{k=1}^{2}a_k\left(\frac{t}{t_w}\right)^{\lambda_k}. \tag{4}$$

The (negative) prefactors and exponents entering the expression are for completeness tabulated in the Appendix.

IV. ORIGIN OF SUBAGING AND END-OF-AGING

Let T denote the isothermal aging temperature. As mentioned, C denotes the smallest relevant relaxation time, i.e., the time associated to the smallest energy barrier in the energy landscape of a single domain. Consider now a domain characterized by the initial barrier b^*, or equivalently, by the Arrhenius time $t^*=C\exp(b^*/T)$. If $t_w<t^*$, the behavior at time $t=t_w$ remains controlled by the initial barrier b^* and the domain's contribution to the TRM correspondingly depends on t/t^*. Conversely, if $t_w>t^*$ the initial barrier has been sur-

mounted at t_w, and the scaling variable is hence t/t_w. In real space, the size of the domain grows as a function of the dynamical barrier $b(t)$. E.g. a power-law growth in the time domain [12] corresponds to an exponential growth in $b(t)$. In our case, $b(t)=\max[T\ln(t/C), b^*]$, and, compared to the case $b^*=0$, domain growth is delayed up to the Arrhenius time t^*.

Returning to the form of the response, the contribution of a domain with initial dynamical barrier b^* is

$$m(t,t_w,b^*) = \eta(t_w - t^*)M_0(t/t_w) + \eta(t^* - t_w)M_0(t/t^*), \quad (5)$$

where η is again the Heaviside function, where M_0 is given in Eq. (3) and where $t \geq t_w$. The formula embodies the key feature of hierarchical relaxation without reference to any specific model. Second, it introduces spatial heterogeneity, as b^*, or equivalently, the Arrhenius time t^*, is allowed to differ across the domains. At $t_w=t^*$, the Heaviside function η switches between the two scaling forms available for the magnetic response of a single domain. If the barriers of the different domains in the system are all initially near zero, only the first term contributes. and the PA behavior given by $M_0(t/t_w)$ goes through at the macroscopic level. In the general case, the TRM decay takes the form,

$$\begin{aligned} M(t,t_w) = M_0(t/t_w)\int_0^{T\ln(t_w/C)} P(b)db \\ + \int_{T\ln(t_w/C)}^{T\ln(t/C)} M_0\left(\frac{t}{C}e^{-b/T}\right)P(b)db \\ + M_I\int_{T\ln(t/C)}^{\infty} P(b)db, \end{aligned} \quad (6)$$

where $P(b)$ is the probability density for a domain with barrier in the initial state.

Inserting Eq. (1) into Eq. (6), and using simple algebraic manipulations, one arrives at

$$\begin{aligned} M(t,t_w) = M_I\left(\frac{t}{C}\right)^{-x} + \left[1 - \left(\frac{t_w}{C}\right)^{-x}\right]M_0(t/t_w) \\ + x\left(\frac{t}{C}\right)^{-x}\int_1^{t/tw} M_0(z)z^{x-1}dz. \end{aligned} \quad (7)$$

Using the parameterization of M_0 given in Eq. (3), the last expression becomes

$$M(t,t_w) = M_0(t/t_w) + \left(\frac{t_w}{C}\right)^{-x}\sum_{i=1}^{2}\frac{a_i}{\lambda_i + x}\left[\left(\frac{t}{t_w}\right)^{-x} - \left(\frac{t}{t_w}\right)^{\lambda_i}\right]. \quad (8)$$

Pure aging is achieved in the limit $x=\infty$, i.e., when all initial barriers are equal to zero with probability one. Subaging is present for *intermediate* values of x. Let now λ_2 be the largest of the two decay exponents characterizing the PA regime. If and only if $x<-\lambda_2$, the asymptotically dominant contribution to the TRM for large t is proportional to $(\frac{t}{C})^{-x}$. To bear this out, we first rewrite our last equation as

TABLE I. The first column contains the ratios of the isothermal aging temperature T, at which the measurements are taken to the critical temperature T_g. The other columns contain the prefactors and exponents of the two power-law terms appearing in Eq. (3). All values are obtained by fits (not shown) of quality similar to Fig. 2.

T/T_g	a_1	λ_1	a_2	λ_2
0.40	−0.0878	−2.7575	−0.0289	−0.1869
0.60	−0.0981	−3.0945	−0.0462	−0.2609
0.83	−0.1365	−2.9016	−0.0527	−0.3102
0.90	−0.1309	−3.2785	−0.0418	−0.3098
0.95	−0.1117	−3.4737	−0.0296	−0.3495

$$M(t,t_w) = M_0(t/t_w) + \left(\frac{t}{C}\right)^{-x}\sum_{i=1}^{2}\frac{a_i}{\lambda_i + x}\left[1 - \left(\frac{t}{t_w}\right)^{\lambda_i + x}\right]. \quad (9)$$

The PA term $M_0(t/t_w)$ decays the fastest and can be neglected. In what remains, the t_w dependent term having the slowest decay is $(t/t_w)^{\lambda_2+x}$. In order for the EOA behavior to set in, this term must be much smaller than one. E.g., a relative deviation of the TRM curve from EOA equal to 1/10 is reached at time

$$t_{\mathrm{EOA}} = (10)^{1/|x+\lambda_2|}t_w. \quad (10)$$

Since $\frac{1}{|x+\lambda_2|}$ is very large when $x+\lambda_2 \approx 0$, the model predicts that EOA may occur on a time scale which diverges very rapidly with t_w. This is qualitatively in accord with the experimental observations of [11]. Second, when the exponent x is numerically small, the expansion $(t/C)^{-x} = 1 - x\ln(t/C) + \mathcal{O}\{[x\ln(t/C)]^2\}$ is applicable, and the TRM decays in a nearly logarithmic fashion for a wide range of t, likewise in accord with the experimental findings.

V. ON $t_{\mathrm{obs}}/t_w^{\mu}$ SCALING

Equation (8) features a clear sub-aging behavior with no reference to the scaling variable t/t_w^{μ}. Model TRM curves generated using the equation can nevertheless be empirically scaled in the traditional manner for an intermediate range of t and t_w values. This is checked numerically (i) by evaluating Eq. (8), with $C=1$, $t_{\mathrm{M}}^*=10$ and with all other parameters given in Table I, and (ii) by scaling the curves obtained as usually done for experimental data. The left panel in Fig. 1 is shown to confirm that t_{obs}/t_w scaling is unsatisfactory for the parameter values utilized: The four curves plotted do not collapse in the midrange of the abscissa. The curves all pertain to $T=0.6T_g$, and correspond to $t_w=50$, 100, 1000, and 10 000. The exact same data are shown in the middle panel of the same figure, now plotted versus $t_{\mathrm{obs}}/t_w^{\mu}$. The value of μ is chosen to optimize the data collapse, which is visibly improved. Additional curves (not shown) were similarly obtained for $T/T_g=0.4, 0.6, 0.9$, and $T=0.93$. The corresponding empirical μ values gauge how close the TRM is to PA. These values are plotted versus T/T_g in the right panel of the same figure (the line is only a guide to the eye). Interestingly,

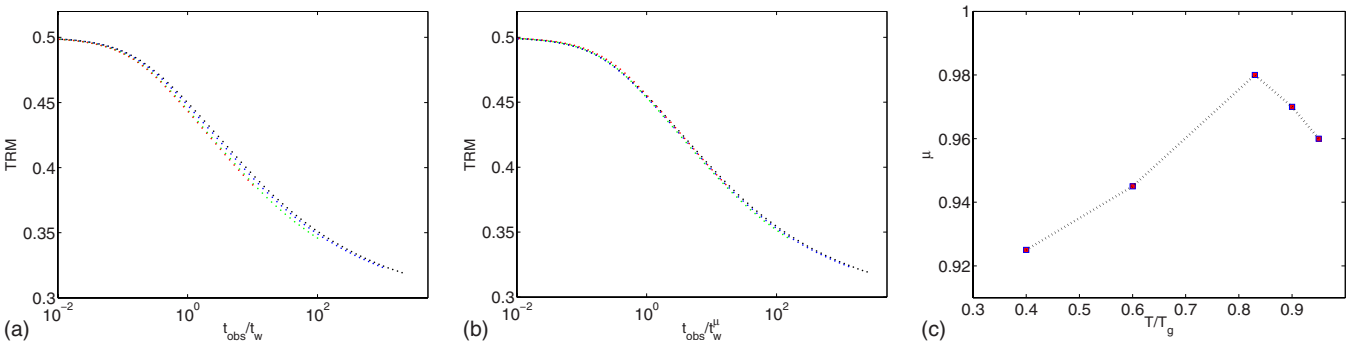

FIG. 1. (Color online) The Thermoremanent Magnetization, calculated according to Eq. (8), is plotted for t_w=50, 100, 1000, and 10 000 versus t_{obs}/t_w, (left panel), and versus t_{obs}/t_w^μ, (middle panel). The left panel shows that a t/t_w scaling does not work satisfactorily. The middle panel shows that the standard t/t_w^μ scaling procedure visibly improves the quality of the data collapse. All data are for $T=0.60T_g$. Similar scaling plots were also obtained for other temperatures (not shown). In each case the exponent μ was estimated as the value providing the best data collapse. In the rightmost panel, the μ values, thus, obtained are plotted versus T/T_g (squares). The dotted line is only a guide to the eye. A clear maximum is visible at T/T_g=0.83.

μ versus T peaks at T/T_g=0.83, the very temperature where μ is experimentally closest to unity [5].

To conclude, Eq. (8) fully contains the standard SA behavior widely seen in spin glasses. Furthermore, it implies that the applicability of t_{obs}/t_w^μ scaling per se does not endow μ with physical significance, since the latter is plainly absent in our case.

VI. SUMMARY AND OUTLOOK

In this work, the known scaling properties of off-equilibrium linear response functions in spin glasses have been accounted for by combining two aspects of complex dynamics: the hierarchical relaxation of independently thermalizing domains, coupled with the spatial heterogeneity of the initial domain configurations, as defined by their initial dynamical barriers. Such barriers would uniformly vanish for PA behavior. Our analysis relies on generic properties of complex dynamics, and should, therefore, be widely applicable to glassy systems with short-range interactions. These might include quantum spin glasses, whose critical behavior has recently been investigated [31,32], and irrespective of whether a true equilibrium phase transition exists [32] or not [31].

The distribution of initial dynamical barriers plays a pivotal role in the theory. Arguably, its form depends on the cooling protocol, e.g., fast cooling could give a distribution more sharply peaked at zero, and lead to a relaxation scaling form closer to PA. The width of the initial barrier distribution is expressed by the exponent x, which is *experimentally accessible* as the logarithmic slope of the TRM decay for very large values of t/t_w, i.e., in the dynamical regime where the t_w dependence of the data is absent. It should therefore be possible to empirically study, via x, how the initial cooling protocol affects the distribution of initial barriers and, indirectly, the subsequent relaxation dynamics.

In this work, dynamical heterogeneity is attached to the thermalized domains, which determine the real-space evolution of spin glasses with short-ranged interactions. While this choice is natural for the case at hand, other interpretations are possible. Specifically, mean field models with no spatial structures can explain features of hole burning experiments [33]. The source of dynamical heterogeneity may hence, in some cases, differ from spatial structures. Our formalism can accommodate different interpretations. Its key feature is superposing, in a linear fashion, the hierarchical dynamics of independent, or nearly independent subsystems each starting out with a different initial conditions. The independence of the subsystems is granted, within a domain interpretations, by separation in space. More generally, a nearly decomposable interaction matrix would provide a similar effect.

APPENDIX

We describe in this Appendix how the parameter values entering Eq. (3) are estimated by fitting to experimental

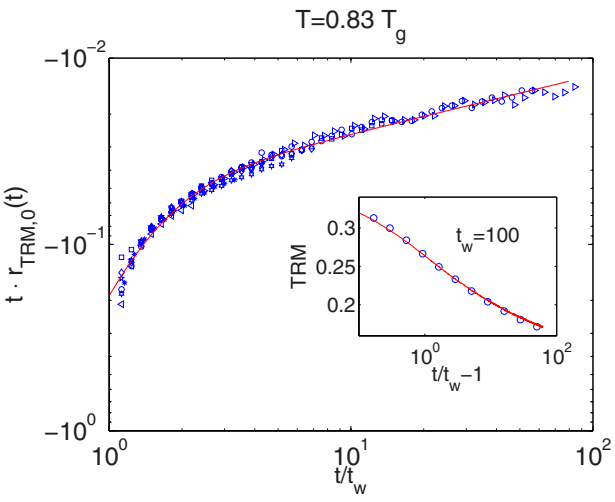

FIG. 2. (Color online) The TRM decay rate, multiplied by the age t is plotted versus t/t_w for T/T_g=0.83 and t_w=50 (right pointing triangles), 100 (circles), 300 (squares), 630 (diamonds), 1000 (pentagrams), 3600 (hexagrams), 6310 (asterisks) and (10 000) (left pointing triangles). The line is given by Eq. (4). The insert compares the TRM decay measured at t_w=100s (red line), with the theoretical estimates (blue circles) obtained by Eq. (3).

TRM data. The data are obtained according to a standard procedure: a $Cu_{0.94}Mn_{0.06}$ spin glass sample is rapidly quenched to a temperature $T<T_g$ in the presence of a small magnetic field. The field is cut at time $t=t_w$, and the magnetization decay is then recorded [2,4–6] for $t>t_w$.

The parameters shown in Table I are used to *empirically determine*, on the basis of Eq. (8), the SA exponent $\mu(T)$. The small deviation of the experimental data from the PA form given in Eq. (3) implies, of course, a small systematic error. Of special importance are the values of the dominant exponent λ_2 which are listed in the last column of the table: According to Eq. (10), the quantity λ_2+x determines the time scale for the onset of EOA behavior.

Figure 2 is scaling plot of the rate of (de-) magnetization multiplied by the system age, versus the scaling variable t/t_w. At the aging temperature $T=0.83$ the empirical subaging exponent μ is very close to one [4,5] and the data conform reasonably well to a PA ansatz. The approximation is worse, but still usable, at other aging temperatures and all data can be fit reasonably well by the full aging formula Eq. (3).

[1] L. C. E. Struik, *Physical Aging in Amorphous Polymers and Other Materials* (Elsevier Science Ltd., New York, 1978).
[2] M. Alba, M. Ocio, and J. Hammann, Europhys. Lett. **2**, 45 (1986).
[3] G. R. K. Reddy and Y. M. Joshi, J. Appl. Phys. **104**, 094901 (2008).
[4] V. S. Zotev, G. F. Rodriguez, G. G. Kenning, R. Orbach, E. Vincent, and J. Hammann, Phys. Rev. B **67**, 184422 (2003).
[5] G. F. Rodriguez, G. G. Kenning, and R. Orbach, Phys. Rev. Lett. **91**, 037203 (2003).
[6] D. Parker, F. Ladieu, J. Hammann, and E. Vincent, Phys. Rev. B **74**, 184432 (2006).
[7] Bernd Rinn, Philipp Maass, and Jean-Philippe Bouchaud, Phys. Rev. Lett. **84**, 5403 (2000).
[8] L. Berthier and J.-P. Bouchaud, Phys. Rev. B **66**, 054404 (2002).
[9] I. S. Suzuki and M. Suzuki, Phys. Rev. B **68**, 094424 (2003).
[10] P. Sibani, G. F. Rodriguez, and G. G. Kenning, Phys. Rev. B **74**, 224407 (2006).
[11] G. G. Kenning, G. F. Rodriguez, and R. Orbach, Phys. Rev. Lett. **97**, 057201 (2006).
[12] J. Kisker, L. Santen, M. Schreckenberg, and H. Rieger, Phys. Rev. B **53**, 6418 (1996).
[13] A. J. Bray and M. A. Moore, Phys. Rev. Lett. **58**, 57 (1987).
[14] G. Parisi, Phys. Rev. Lett. **50**, 1946 (1983).
[15] Y. G. Joh, R. Orbach, and J. Hammann, Phys. Rev. Lett. **77**, 4648 (1996).
[16] H. A. Simon, Proc. Am. Philos. Soc. **106**, 467 (1962).
[17] P. Sibani, C. Schön, P. Salamon, and J.-O. Andersson, Europhys. Lett. **22**, 479 (1993).
[18] P. Sibani and P. Schriver, Phys. Rev. B **49**, 6667 (1994).
[19] S. Grossmann, F. Wegner, and K. H. Hoffmann, Phys. Lett. **46**, 575 (1985).
[20] Paolo Sibani and Karl Heinz Hoffmann, Phys. Rev. Lett. **63**, 2853 (1989).
[21] K. H. Hoffmann, S. Schubert, and P. Sibani, Europhys. Lett. **38**, 613 (1997).
[22] R. G. Palmer, Adv. Phys. **31**, 669 (1982).
[23] P. Sibani and P. B. Littlewood, Phys. Rev. Lett. **71**, 1482 (1993).
[24] Paolo Sibani and Jesper Dall, Europhys. Lett. **64**, 8 (2003).
[25] P. Sibani and H. Jeldtoft Jensen, Europhys. Lett. **69**, 563 (2005).
[26] A. Fischer, K. H. Hoffmann, and P. Sibani, Phys. Rev. E **77**, 041120 (2008).
[27] E. Vincent, in *Recent Progress in Random Magnets*, edited by D. H. Ryan (McGill University, Montreal, 1991), pp. 209–246.
[28] K. Jonason, E. Vincent, J. Hammann, J. P. Bouchaud, and P. Nordblad, Phys. Rev. Lett. **81**, 3243 (1998).
[29] R. V. Chamberlin, Phys. Rev. Lett. **83**, 5134 (1999).
[30] G. G. Kenning, J. Bowen, P. Sibani, and G. F. Rodriguez, e-print arXiv:0910.1924.
[31] P. E. Jonsson, R. Mathieu, W. Wernsdorfer, A. M. Tkachuk, and B. Barbara, Phys. Rev. Lett. **98**, 256403 (2007).
[32] C. Ancona-Torres, D. M. Silevitch, G. Aeppli, and T. F. Rosenbaum, Phys. Rev. Lett. **101**, 057201 (2008).
[33] L. F. Cugliandolo and J. L. Iguain, Phys. Rev. Lett. **85**, 3448 (2000).